ECHO-TEXTURE ANALYSIS FOR SONAR INFORMATION PROCESSING

Douglas A. Abraham
CausaSci LLC
P.O. Box 627
Ellicott City, MD 20141
USA

ABSTRACT
Echo texture, as defined by the K-distribution shape parameter, is evaluated as a feature useful in sonar information processing for the separation of targets and clutter. A shifted gamma distribution is proposed for representing the statistics of the logarithm of the texture estimator and seen to provide a very good fit for examples from exponentially-, Weibull-, and K-distributed data and an adequate fit for a very heavy-tailed generalized Pareto example. Empirical fits to the moment statistics (mean, standard deviation, and coefficient of skewness) of the log-texture for simulated data allow evaluation of the texture distributions for data-sample sizes ranging from 50 to 1,000 independent samples. Single-feature pairwise classification performance is evaluated, illustrating the improvement attainable through the use of additional data in estimating texture.

Index Terms—K distribution, clutter, texture, estimation, classification

1. INTRODUCTION AND BACKGROUND
The texture of an active sonar echo can describe the statistical character of the backscattering and has the potential for separating clutter-source echoes from target-like backscattering having a similar echo-to-background power ratio but different texture [1, 2]. In assessing the texture of an echo, the K-distribution [3] shape parameter is a useful descriptor as it has a direct link to scattering physics [4]. The focus of this paper is on characterizing the statistics of an echo-texture estimator for use in active sonar information processing (i.e., tracking and classification).

Active sonar signal processing typically entails beamforming, matched filtering, and detection, resulting in contact-level data. In many sonar systems, the cross-range resolution is poor enough that the useful data reflected from spatially compact objects are restricted to one beam and can be represented as a time-series snippet extracted about the central range of the contact-level data. After estimating the start and stop time of the echo (see e.g., [5, 6]), define the time-series at the matched-filter intensity level as

$$Y = [Y_1 \cdots Y_n]^T,$$

(1)

where \(n\) is the number of independent samples spanning the echo. Statistical independence is approximately achieved by sampling every \(T\) seconds where \(T\) is at least one over the transmit waveform bandwidth.

Information processing in active sonar fulfills the localization, tracking and classification objectives through the use of contact-level data and associated snippet of echo time-series. Statistical measures formed from the echo time series are used in feature-aided tracking to improve contact-to-track association and in classification to enable separation between clutter and target classes.

In Sect. 2, the benefits of using the shape parameter of the K distribution to represent echo texture and for use as a tracking or classification feature are presented along with an estimation technique. A shifted-gamma probability density function (PDF) is proposed in Sect. 3 for representing the statistical distribution of the logarithm of the K-distribution shape parameter estimate with the goodness of fit evaluated for a variety of simulated data examples spanning a Rayleigh-distributed envelope to a very heavy-tailed generalized-Pareto data example. Finally, in Sect. 4, empirically derived models for the echo-texture estimator moment statistics are used with the shifted-gamma model to evaluate pairwise classification performance as a function of the number of independent samples used in the texture estimation.

2. ECHO TEXTURE: THE K-DISTRIBUTION SHAPE PARAMETER
The K distribution [3] has been applied in both radar and sonar to represent data with heavy-tailed PDFs. The PDF of the distribution for matched-filter intensity data is

$$f_K(y) = \frac{2}{\Gamma(\alpha)} \left(\frac{y}{\lambda}\right)^{\alpha-1} \frac{1}{\sqrt{2\pi}} K_{\alpha-1} \left(2\sqrt{\frac{y}{\lambda}}\right),$$

(2)
for $y > 0$ where $\alpha > 0$ is the shape parameter, $\lambda > 0$ is the scale, and the average intensity is $\alpha \lambda$. A K-distributed intensity tends to an exponential distribution (i.e., Rayleigh envelope) as the shape parameter $\alpha \to \infty$, with smaller values of $\alpha$ representing heavier tailed distributions.

### 2.1. Characterization of scattering physics

The K distribution is of interest not only because it is capable of characterizing realistic heavy-tailed sonar data, but also because of its representation of scattering physics. Scattering from a sub-fractal rough surface [7] or from a finite number of scatterers with exponentially distributed sizes (e.g., cross-sections) [4] leads directly to K-distributed backscattering. For the latter situation, the shape parameter $\alpha$ is equal to half the number of independent scatterers in the sonar resolution cell. Many, though not all, sources of clutter are expected to be separable from target echoes based on their number of independent scattering centers.

### 2.2. Invariances

In choosing a statistical feature for use in sonar information processing, the first choice would be for a sufficient statistic containing all the relevant information of the full data sample, but with a much smaller dimension. As this is a difficult task, the feature set is usually chosen empirically emphasizing features that are statistically independent and invariant to nuisance parameters.

The K-distribution shape parameter, as estimated using the approach described in Sect. 2.3, is invariant to scale within the analysis sample $Y$. It was also shown to be insensitive to the multipath effects of acoustic propagation [8] when the scattering source is spatially compact. Finally, the effect of a varying echo-to-background noise power ratio (EBR) can be mitigated through a simple correction found in [6] that expands on that shown in [9] and illustrates the effect using real data. Independence, or at least some moderate decoupling, from EBR is important as it implies the feature contains information not already exploited in making the detection decision, which is essential in discarding false alarms arising from very heavy-tailed clutter or discrete clutter events (i.e., clutter discrete).

As described in [6], if $\hat{\alpha}_{e+b}$ is the shape parameter estimated from the sample $Y$ containing both the echo of interest and background noise or diffuse reverberation, then the shape parameter estimate for the echo portion of the data is

$$\hat{\alpha}_e = \frac{\hat{\alpha}_{e+b}}{\left(1 + \frac{1}{s_e}\right)^4} \quad (3)$$

$$= \hat{\alpha}_{e+b} \left(\frac{\hat{P}_{e+b} - \hat{P}_b}{\hat{P}_{e+b}}\right)^4 \quad (4)$$

where the EBR,

$$s_e = \frac{\hat{P}_{e+b}}{\hat{P}_b} - 1, \quad (5)$$

is formed from the average power in the analysis window

$$\hat{P}_{e+b} = \frac{1}{n} \sum_{i=1}^{n} Y_i \quad (6)$$

and an estimate of the background power ($\hat{P}_b$) from auxiliary data in nearby ranges or beams.

### 2.3. Estimation

Many algorithms have been proposed for estimating the shape parameter of the K distribution [10, 11, 12] including maximum-likelihood, method-of-moments, expectation-maximization, and mixed methods. Maximum-likelihood approaches are generally plagued by the need to evaluate the K-Bessel function, while method-of-moment approaches usually have a non-zero probability of resulting in non-invertible moment equations [13].

The technique used in this analysis is a Bayesian approach developed in [14] based on the method of moments estimator described in [10] using the first and second moments of the matched-filter envelope data. Define the parameter $D$ as the non-linear mapping of $\alpha$,

$$D = \left[\frac{4\alpha \Gamma^2(\alpha)}{\pi \Gamma^2(\alpha + 1/2)} - 1\right]^{-1}. \quad (7)$$

As $\alpha \to \infty$, $D \to d_{\text{max}} = \pi/(4 - \pi)$ from below such that $D$ is always $< d_{\text{max}}$ for finite values of $\alpha$. The method-of-moments approach described in [10] approximates $D$ by the ratio of the square of the sample mean on the envelope to the sample variance,

$$T = \frac{\bar{X}^2}{\sigma_X^2}, \quad (8)$$

where

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} \sqrt{Y_i} \quad (9)$$

is the sample mean and the sample variance is

$$\sigma_X^2 = \frac{1}{n} \sum_{i=1}^{n} \left(\sqrt{Y_i} - \bar{X}\right)^2. \quad (10)$$

The moment equation is simply (7) with $D$ set to $T$ and is invertible when $T$ is less than $d_{\text{max}}$. As described in [14], the estimation failure occurring when $T \geq d_{\text{max}}$ is avoided by approximating the likelihood function of $D$ by a gamma PDF with parameters obtained from the data. The shape parameter estimate is then formed by inverting (7) using the posterior mean of $D$ with a prior requiring that $D \leq d_{\text{max}}$ so the mapping is invertible.
The moment ratio of (8) is the same one used in the amplitude contrast measure of [1] where it was seen to provide the best balance between sensitivity to the shape parameter value and robustness to additive noise. While the results presented here might be similar to those achieved by using the straight method-of-moments estimator detailed in [10], the non-zero probability of non-invertible moment equations will skew them.

Owing to the importance of the parameter estimation to the theme of this paper, the estimator is described in enough detail to allow implementation in Appendix A.

3. STATISTICAL MODEL FOR THE TEXTURE PARAMETER ESTIMATE

Active sonar information processing functions such as target tracking and classification require knowledge of the statistical properties of the features they use. In statistical classifiers, this often requires using training data to estimate the joint PDF over all the features or the use of marginal PDFs and a Copula function [15] to account for the inter-feature dependencies.

One of the primary contributions of this paper is the identification of a statistical model approximating the distribution of the K-distribution shape parameter estimate derived in [14] and described in Sect. 2.3 and Appendix A.

3.1. Shipwreck data and the log-normal distribution

An analysis of active sonar echoes from a shipwreck presented in [6] illustrated the efficacy of the log-normal distribution in describing the shape-parameter estimate statistics. The cumulative distribution function (CDF) of the shape parameter estimates obtained over a wide range of aspect angles from a monostatic sonar at 2-km and 10-km ranges and from a bistatic sonar at 12-km range as presented in [6] is shown here in Fig. 1 along with the log-normal CDF using parameters estimated from the data. The shape parameter estimates were corrected for varying EBR as described in [6] and (4) and very strongly support the log-normal distribution as an appropriate statistical model. The log-normal PDF has the form [16, Ch. 26]

\[
 f_{\hat{\alpha}} (\hat{\alpha}; \sigma) = \frac{1}{\hat{\alpha}\sigma\sqrt{2\pi}} e^{-[\log(\hat{\alpha})-\mu]^2/(2\sigma^2)}, \tag{11}
\]

for \( \hat{\alpha} > 0 \) where \( \mu \) and \( \sigma \) are the mean and standard deviation of the logarithm of \( \hat{\alpha} \).

The data indicate the wreck echoes have a shape parameter of approximately 2.5; however, the log-normal shape of the PDF might arise from either a natural variation in the scattering statistics with aspect angle on the wreck or from the randomness of the parameter estimation procedure. To resolve this issue, shape-parameter estimates are formed from simulated K-distributed data with \( \alpha = 2.5 \) and fitted to a log-normal model predicted for a sample size of \( n = 164 \) (which was the average number of independent samples in the wreck echoes) as described in the Sect. 3.2. As seen in Fig. 1, the prediction shows a slight bias toward a higher shape parameter and less variance than the real data. The former arises from not accounting for the estimator bias when interpreting the real-data results while the higher variance of the real data might arise from changes in the scattering from the wreck over the large span of aspect angles from which the data were obtained (covering approximately 74 degrees) or from having to estimate the texture using data corrupted by additive noise. Despite these minor differences, it is clear that the randomness of the estimation process is the dominant factor in inducing the near log-normal PDF on the texture estimator for the shipwreck data.

3.2. The shifted-gamma model for the logarithm of texture

While the log-normal distribution appears adequate for modeling the texture parameter estimate statistics, histograms formed on the log-texture (i.e., \( \log(\hat{\alpha}) \)) from simulated data exhibit some skew, while the log-normal model (which is normally distributed on the \( \log(\hat{\alpha}) \) domain) is symmetric. As such, a better fit can be obtained by using a shifted gamma distribution in the \( \log(\hat{\alpha}) \) domain. The shifted-gamma PDF is simply

\[
 f_G(z) = \frac{(z-z_0)^{\gamma-1}}{\Gamma(\gamma)\beta^\gamma} e^{-(z-z_0)/\beta} \tag{12}
\]
for \( z \geq z_0 \) where \( z = \log \alpha \), \( z_0 \) is the shift parameter, \( \gamma \) is the shape, and \( \beta \) is the scale. The parameters may be obtained through moment matching according to

\[
z_0 = \mu - 2 \frac{\sigma}{\eta_3}
\]

(13)

\[
\gamma = \frac{4}{\eta_3}
\]

(14)

and

\[
\beta = \frac{\sigma \eta_3}{2}
\]

(15)

where \((\mu, \sigma, \eta_3)\) are the mean, standard deviation, and coefficient of skewness. Note that when \( \beta \) is negative, the distribution flips to cover the range \( z \in (-\infty, z_0] \).

The PDF of the shifted-gamma distribution when converted to the texture domain \((\hat{\alpha} = e^Z)\) is

\[
f_{\hat{\alpha}}(\hat{\alpha}) = \frac{\alpha_0}{\beta \Gamma(\gamma)} \left( \frac{\hat{\alpha}}{\alpha_0} \right)^{\frac{1}{\beta} - 1} \left[ \frac{1}{\beta} \log \left( \frac{\hat{\alpha}}{\alpha_0} \right) \right]^{\gamma - 1}
\]

(16)

for \( \hat{\alpha} > \alpha_0 \) where \( \alpha_0 = e^{\eta_0} \). When \( \beta < 0 \), the distribution has support over \( \hat{\alpha} \leq \alpha_0 \). Similar to the log-normal, this distribution will be referred to as log-shifted-gamma where taking the logarithm of \( \hat{\alpha} \) results in a shifted-gamma distribution.

### 3.3. Empirical fits for various data examples

To evaluate when the shifted-gamma PDF is a good statistical model for the logarithm of the texture parameter estimates and generate the moment statistics (mean, standard deviation, and coefficient of skewness) necessary for evaluation of classification performance, a simulation analysis is performed where 100,000 data sets are generated under four different data models and used to estimate the K-distribution shape parameter. The four data models, as described in Table 1, start with the standard exponential intensity (i.e., a Rayleigh-distributed envelope) and progress in tail heaviness with Weibull-, K-, and generalized-Pareto-distributed examples. The shape-parameter values chosen for each distribution are shown in Table 1 and the \( P_{fa} \) curves in Fig. 2. The K-distribution PDF was presented in (2); the PDFs for the other data examples are found in Appendix B. Owing to the scale invariance of \( \hat{\alpha} \), the power or scale of the distributions has no impact on this analysis. The shipwreck data are emulated in the K-distribution example by choosing a shape parameter of \( \alpha = 2.5 \). The generalized-Pareto (GP) example is extremely heavy tailed and might be representative of a target echo while the Weibull example represents more diffuse scattering than the shipwreck (e.g., an exposed ridge).

Histogram CDFs of the texture parameter estimate and corresponding log-shifted-gamma model are shown in Fig. 3 where the fits for the exponential, Weibull, and K data are exceptional with almost no difference visible on this scale.

### Table 1. Shape parameters for the data-distribution examples evaluated.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Shape parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>n/a</td>
</tr>
<tr>
<td>Weibull</td>
<td>( \beta = 0.85 )</td>
</tr>
<tr>
<td>K</td>
<td>( \alpha = 2.5 )</td>
</tr>
<tr>
<td>GP</td>
<td>( \gamma = 0.5 )</td>
</tr>
</tbody>
</table>

**Fig. 2.** \( P_{fa} \) for the data-distribution examples evaluated.

**Fig. 3.** Histogram CDFs of \( \hat{\alpha} \) (solid lines) and shifted-gamma fits (dashed lines) for \( n = 500 \). The fits for the K, Weibull, exponential data models are nearly identical to the histograms.
The GP data however, present a clear mismatch between the data and the log-shifted-gamma model. Although the disparity for the lighter-tailed data appears minimal, none of these fits would be acceptable at a reasonable $p$-value as evaluated through a goodness of fit test using all 100,000 estimates. However, the fit is accurate enough that it is useful in sonar information processing algorithms such as tracking and classification.

Future research will endeavor to analytically link the shape parameter of the data distribution and the data-sample size ($n$) to the parameters of the log-shifted-gamma PDF for the texture estimator $\hat{\alpha}$ through moment matching. However, noting the complicated form of the estimator and the need for third-order moments, a simpler approach is taken in this paper to obtain the dependency on $n$ at the expense of not accounting for the shape-parameter dependency. In addition to the $n = 500$ case presented in Fig. 3, several other data-sample sizes ranging from 50 to 1,000 were simulated and evaluated. For each value of $n$, the sample mean, standard deviation, and coefficient of skewness of $\log \hat{\alpha}$ were obtained using all 100,000 trials. The parameter values as a function of sample size $n$ were then fit using a least-squared-error (LSE) optimization to a model with a constant term and dependency on $1/\sqrt{n}$, $1/n$, and $\log n$. Each parameter ($\mu$, $\sigma$, or $\eta_3$) can then be approximated by using the coefficients shown in Tables 2–4 and the equations:

$$\mu(n) \approx a_0 + \frac{a_1}{\sqrt{n}} + \frac{a_2}{n} + a_3 \log n \quad (17)$$

$$\sigma(n) \approx b_0 + \frac{b_1}{\sqrt{n}} + \frac{b_2}{n} + b_3 \log n \quad (18)$$

$$\eta_3(n) \approx c_0 + \frac{c_1}{\sqrt{n}} + \frac{c_2}{n} + c_3 \log n. \quad (19)$$

In obtaining the coefficients in Tables 2–4, the squared error was evaluated over all the different combinations of dependencies (e.g., choosing different combinations from 1, $1/\sqrt{n}$, $1/n$, and $\log n$) while restricting the coefficients to three digits of accuracy. The estimated and fitted values are displayed in Figs. 4–6 where, on these scales, the fits are quite accurate with little visible disparity for the mean and standard deviation and minor deviation for the coefficient of skewness.

Fig. 6 illustrates the non-zero skewness of the log-texture estimator for the various data models with the least occurring in the exponentially distributed data and the most for the GP data. The minimal skewness in the log-texture estimate for the exponentially distributed data, coupled with the high values of the gamma shape parameter seen in Fig. 7, indicates that the log-normal model proposed in [6] should be quite good for the texture estimator. Although not shown here, this was confirmed by a goodness-of-fit analysis similar to that shown in Sect. 3.4 for the shifted-gamma model. The normal model also provided adequate fits for the log-texture estimates from Weibull and K data, though not as good as the shifted-gamma distribution.

In this paper, (17)–(19) are used with the coefficients found in Tables 2–4 to obtain the parameters of the shifted-gamma distribution predicted for a given value of $n$ for the example data distributions described in Table 1. They may also be used to form predictions for other models (e.g., the log-normal as shown in Fig. 1), although the accuracy of the fit would still require evaluation. The coefficients found in Tables 2–4 are for specific values of the shape parameter of the heavy-tailed distributions considered, so are primarily useful in the prediction of performance as presented in Sect. 4. As the exponential data model has no shape parameter, these coefficients may be useful with (17)–(19) in representing the texture-estimate statistics as a function of sample size when a Rayleigh-distributed-envelope hypothesis is necessary. Clearly use of these fits outside of the $n \in [50, 1000]$ range or for other values of the data-distribution shape parameters can or should yield inaccurate approximations.

### Table 2. Coefficients for $\mu$.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>$-0.277$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0.503$</td>
</tr>
<tr>
<td>Weibull</td>
<td>$2.16$</td>
<td>$-3.65$</td>
<td>$-6.10$</td>
<td>$-0.111$</td>
</tr>
<tr>
<td>K</td>
<td>$0.972$</td>
<td>$2.28$</td>
<td>$-18.3$</td>
<td>$-0.0142$</td>
</tr>
<tr>
<td>GP</td>
<td>$-0.628$</td>
<td>$7.46$</td>
<td>$-18.9$</td>
<td>$0.0668$</td>
</tr>
</tbody>
</table>

### Table 3. Coefficients for $\sigma$.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>$b_0$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>$0.296$</td>
<td>$0.897$</td>
<td>$-1.12$</td>
<td>$0.00519$</td>
</tr>
<tr>
<td>Weibull</td>
<td>$1.03$</td>
<td>$0$</td>
<td>$-5.29$</td>
<td>$-0.122$</td>
</tr>
<tr>
<td>K</td>
<td>$0.415$</td>
<td>$3.92$</td>
<td>$-14.3$</td>
<td>$-0.0541$</td>
</tr>
<tr>
<td>GP</td>
<td>$-0.230$</td>
<td>$8.94$</td>
<td>$-26.4$</td>
<td>$0.0333$</td>
</tr>
</tbody>
</table>

### Table 4. Coefficients for $\eta_3$.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>$c_0$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>$-0.277$</td>
<td>$2.63$</td>
<td>$-11.0$</td>
<td>$0.00184$</td>
</tr>
<tr>
<td>Weibull</td>
<td>$7.76$</td>
<td>$-50.2$</td>
<td>$141$</td>
<td>$-0.841$</td>
</tr>
<tr>
<td>K</td>
<td>$7.59$</td>
<td>$-41.8$</td>
<td>$98.4$</td>
<td>$-0.878$</td>
</tr>
<tr>
<td>GP</td>
<td>$0.338$</td>
<td>$16.0$</td>
<td>$-82.4$</td>
<td>$-0.312$</td>
</tr>
</tbody>
</table>

### 3.4. Quality of the approximation

As seen in Fig. 3, the log-shifted-gamma model for the distribution on $\hat{\alpha}$ is an approximation, which raises the question of how good is the fit? To assess the disparity between the model and the observations, the J divergence [17, pg. 117],

$$d_J = \int_{-\infty}^{\infty} \left[ f_1(z) - f_0(z) \right] \log \left( \frac{f_1(z)}{f_0(z)} \right) dz, \quad (20)$$
Fig. 4. Empirical fits to $\mu$ (solid lines) compared with those estimated from simulation (x’s).

Fig. 5. Empirical fits to $\sigma$ (solid lines) compared with those estimated from simulation (x’s).

Fig. 6. Empirical fits to $\eta_3$ (solid lines) compared with those estimated from simulation (x’s).

Fig. 7. Empirical fits to the gamma shape parameter (solid lines) compared with those estimated from simulation (x’s).
between the estimated (histogram) and predicted PDFs of the log-texture is evaluated and \( \sqrt{d_J} \) plotted in Fig. 8 for each of the data examples. If two distributions are normally distributed with the same variance and only differ in the mean, then the square-root of the J divergence is equal to the number of standard deviations that separate the two means. Thus, the values of \( \sqrt{d_J} \) found in Fig. 8 may be interpreted as a disparity in PDF that is similar to an equivalent disparity in terms of the number of standard deviations of difference in the mean for equi-variance Gaussian data. With the similarity between the gamma and normal distributions, especially for high shape parameters in the former, this is a sensible interpretation.

As seen in Fig. 8, the shifted-gamma model is quite good at representing the log-texture estimator statistics from the exponentially-, Weibull- and K-distributed data examples with \( \sqrt{d_J} \) under 0.1 for all sample sizes evaluated. The GP data, with the heaviest tails of those evaluated, had the poorest fits, but was still within about a 0.3-\( \sigma \) difference, which should be adequate for use in most information processing algorithms. The histogram PDF estimate of the log-texture estimate for the GP data example found in Fig. 9 for \( n = 500 \) illustrates why the shifted gamma model has difficulty with this case. The histogram exhibits a large, negative skew and tails on each side of the mode, neither of which can be matched by the gamma model. A better fit might be obtained by differencing two gamma random variables, though this results in a more complicated parameter estimation and PDF evaluation compared with the shifted-gamma model.

4. USING TEXTURE IN SONAR INFORMATION PROCESSING

Post-detection information processing in sonar systems incorporates alternative information such as motion, Doppler, or statistical features to identify and reject false alarms while retaining target echoes. While echo texture (as defined by the K-distribution shape parameter) is not expected to solve all false-alarm problems, it may allow the separation of certain types of clutter from targets. Statistical models like those developed in Sect. 3 can be used to improve association in probabilistic tracking algorithms (e.g., the probabilistic multiple-hypothesis tracker [18]) or be used as marginal densities in a multiple-feature statistical classifier exploiting Copula functions to account for inter-feature dependence [15]. While a single feature evaluated on one echo does not typically provide the necessary separation, accumulation of data or estimates over consecutive pings along a track has the potential to provide significant separation of the echoes solely based on texture.

Of the data distributions considered in Sect. 3, the GP example was chosen as a surrogate target model while the K-distributed example might be target-like or represent a clutter-discrete. The Weibull and exponential examples represent lighter-tailed clutter or background-like data. As seen in Fig. 9, the PDFs of the log-texture array out with smaller values being more target-like. While there is minimal overlap in the PDFs between the GP example and the exponential-data texture, there is significant overlap between the K and Weibull examples. In this section, the classification performance for pairwise comparison between the various examples is examined with an evaluation of how additional data can improve separation.

4.1. Classifier operating characteristics

Classification performance based on the K-distribution shape parameter estimate can be evaluated on either the texture or log-texture domain. In either case, the pairwise evaluation is similar to the evaluation of detection performance with the probability of correct and false classification

\[
P_{cc} = P \{ Z < h | H_1 \} \tag{21}\]

and

\[
P_{fc} = P \{ Z < h | H_0 \} \tag{22}\]

where \( Z = \log \hat{\alpha} \) represents the log-texture domain, \( h \) is a decision threshold and the two models are represented by the hypotheses \( H_0 \) and \( H_1 \) with, respectively, shifted-gamma parameters \((z_0, \gamma_0, \beta_0)\) and \((z_1, \gamma_1, \beta_1)\). For the shifted-gamma model,

\[
P_{cc} = U (-\beta_1) + \text{sign}(\beta_1) F_G \left( \frac{h - z_1}{\beta_1}; \gamma_1 \right) \tag{23}\]
and

\[ P_{fc} = U(-\beta_0) + \text{sign}(\beta_0) F_G\left(\frac{h - z_0}{\beta_0}, \gamma_0\right) \]  

(24)

where \( U(\cdot) \) is the unit step function and \( F_G(h; \gamma) \) is the gamma CDF with shape \( \gamma \) and unit scale.

Varying the decision threshold \( h \) produces the classifier operating characteristic (CLOC) curves as shown in Fig. 10 for \( n = 100 \). The GP-, K-, and Weibull-example textures are easily separated from the texture of the exponential data (solid lines) as is the GP texture from that of the K or Weibull examples. However, the K- and Weibull-example textures are not very separable when only 100 data samples are available for texture estimation. It should be noted that the exponential data are not necessarily from diffuse reverberation or noise-limited backgrounds, but might arise from spatially compact clutter with a high clutter-to-background power ratio and dense scattering (e.g., the bottom-like examples analyzed in [19].)

### 4.2. Improvement with sample size

The models developed in Sect. 3 allow evaluation of single-feature classification performance as a function of the number of independent data samples used in the estimation of echo texture. Notwithstanding the effects of multipath propagation, the number of independent data samples available for texture estimation is approximately the echo duration times the transmit waveform bandwidth (e.g., a 100 ms echo with 1000 Hz of bandwidth yields \( n = 100 \) independent samples). More data for estimation may be obtained by increasing the transmit waveform bandwidth or by combining data from multiple pings and/or receivers. Each approach assumes the scattering statistics are similar under the various situations and may require accounting for different EBR values.

The improvement in performance obtained by increasing the number of data samples is illustrated in Fig. 10 for a fixed \( P_{fc} = 0.05 \) and the various pairwise combinations. While the separability of the K- and Weibull-example textures was poor at \( n = 100 \), a \( P_{cc} = 0.5 \) can be obtained by increasing the estimation data to \( n = 570 \) samples.

![Fig. 9. Histogram PDFs (solid lines) of log \( \hat{\alpha} \) and predicted shifted-gamma fits (dashed lines) for \( n = 500 \).](image)

![Fig. 10. CLOC curves for \( n = 100 \).](image)

![Fig. 11. \( P_{cc} \) improvements with sample size \( (n) \) for \( P_{fc} = 0.05 \).](image)
5. ACKNOWLEDGEMENTS

The data presented in Sect. 3.1 were made possible by the CLUTTER Joint Research Project, including as participants the NATO Undersea Research Centre (NURC), Pennsylvania State University - ARL-PSU (USA), Defence Research and Development Canada - DRDC-A (CAN), and the Naval Research Laboratory - NRL (USA). The author wishes to acknowledge the efforts of all the participants in the Clutter 2007 experiment, especially Scientists in Charge P. Nielsen on R/V Alliance and J. Preston on R/V Oceanus, and experiment collaborators C. Holland and M. Prior.

6. CONCLUSIONS

Statistical models for the texture estimate of an active sonar echo were developed for examples from four different data distributions (exponential, Weibull, K, and generalized Pareto). A shifted-gamma distribution was seen to provide a very good fit to the log-texture for all but the generalized Pareto data, with the exponential log-texture being nearly normally distributed. The texture statistics illustrate that it is easy to separate heavy-tailed data from exponential, even when the latter arise from compact scatterers and high SNR. However, as expected, it is difficult to separate very heavy-tailed data from heavy-tailed data (e.g., a surrogate target from a shipwreck) with small quantities of data.

A least-squared error fitting of the log-texture moment statistics (mean, standard deviation, and coefficient of skewness) allowed interpolation to arbitrary data-sample sizes in the range $n \in [50, 1000]$ for the various examples, enabling evaluation of classification performance as a function of the amount of data available for texture estimation. The statistical models will enable probabilistic feature-aided tracking, potentially improving classification performance by providing explicit marginal distributions for the texture feature, and provide a means for predicting active sonar information-processing performance as a function of bandwidth and when fusing data over multiple pings or distributed receivers.

While the current work was restricted to specific examples from the K, Weibull and GP data distributions, future research will aim to generalize the results to include a dependence on the shape parameter of the input data.

7. REFERENCES


A. SHAPE-PARAMETER ESTIMATION

The information necessary for implementation of the K-distribution shape parameter estimator derived in [14] is presented in this appendix.

Given the matched-filter intensity data from (1), the sample mean and variance of the envelope data ($X_i = \sqrt{Y_i}$) are formed as described in (9) and (10) along with the sample coefficient of kurtosis,

$$\hat{\eta}_4 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^4$$

and used to approximate the moments of the likelihood distribution on $D$,

$$m_1 = \frac{\bar{X}^2}{\sigma^2} \left( 1 - \frac{1}{n} \right) + \frac{1}{n}$$

and

$$m_2 = \frac{\bar{X}^4}{\sigma^2} \left[ 1 + \left( \frac{\hat{\eta}_4 - 3}{n} \right) + \frac{(5 - 2\hat{\eta}_4)}{n^2} \right]$$

$$+ \frac{6\bar{X}^2}{n\sigma^2} \left( 1 - \frac{1}{n} \right) + \frac{3}{n^2}$$

The parameters of the gamma distribution approximating the likelihood function of $D$ are then obtained by moment matching, resulting in the shape parameter

$$a_t = \frac{m_2 - m_1^2}{m_1}$$

and scale parameter

$$b_t = \frac{m_2}{m_1}.$$  \hspace{1cm} (29)

The posterior mean of $D$ given that it is less than the theoretical maximum value of $d_{\text{max}}$ is then

$$\hat{D} = a_t b_t \frac{F_G(d_{\text{max}}; a_t + 1, b_t)}{F_G(d_{\text{max}}; a_t, b_t)}$$ \hspace{1cm} (30)

where $F_G(d; a, b)$ is the gamma CDF with shape $a$ and scale $b$. The K-distribution shape parameter estimate is then obtained by inverting the non-linear function found in (7) after replacing $D$ by $\hat{D}$. As described in [14], the approximate inversion

$$\alpha \approx \frac{1}{4\log \left( \frac{(1 + 1/D) \pi/4}{1} \right)}$$ \hspace{1cm} (31)

is quite accurate for $\alpha \geq 10$ and otherwise useful in starting a Newton-Raphson iteration to obtain a more precise inversion.

B. SIMULATED DATA PDFS

Of the data distributions evaluated in Sect. 3.3 the PDFs of the exponential, Weibull and GP distributions were not defined. The equations below define the PDF for the matched-filter intensity $y \geq 0$ where $\lambda$ is a scale parameter. The exponential PDF is simply

$$f_{E}(y; \lambda) = \frac{1}{\lambda} e^{-\frac{y}{\lambda}}.$$ \hspace{1cm} (32)

The Weibull PDF is

$$f_{W}(y; \beta, \lambda) = \frac{\beta y^{\beta-1}}{\lambda^\beta} e^{-(y/\lambda)^\beta},$$ \hspace{1cm} (33)

where $\beta \in (0, 1]$ is a shape parameter. The GP PDF is

$$f_{GP}(y) = \frac{1}{\lambda \left( 1 + \frac{y}{\lambda} \right)^{\gamma+1}},$$ \hspace{1cm} (34)

where $\gamma$ is a shape parameter.