SPATIALLY VARIANT APODIZATION FOR CONVENTIONAL AND SPARSE SPECTRAL SENSING SYSTEMS

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ABSTRACT

A brief overview of spatially variant apodization (SVA), and its origins as a computationally efficient adaptive windowing technique for synthetic aperture radar (SAR) image formation, is provided by way of introduction. This is followed by a discussion of the use of SVA in conventional (non-sparse) spectral sensing applications such as early warning (EW) and pulsed Doppler radar systems to: (i) reduce the classic window loss in the detection process; (ii) provide enhanced detection of small targets that are close in frequency to large targets. Finally the use of SVA as one element in the processing of sparse multi-coset spectral sensing data is considered. A multi-realisation version of SVA is used to process the parallel uniformly-sub-sampled channels from the analogue to digital converters (ADC’s). The resultant folded high-resolution spectrum is used to make initial detection decisions. The spectrum is then unfolded in the vicinity of these detections using a standard compressive sensing (CS) greedy algorithm known as orthogonalized least squares (OLS). The advantages of this approach are (i) initial detection decisions are based on conventional FFT processing; (ii) CS techniques are only invoked if initial detections are made in (i); (iii) when CS techniques are involved it involves multiple but small CS problems (small in the sense that the number of atoms in each dictionary is given by the largest foldover number possible and the size of each atom is given by the number of ADC’s); (iv) the CS algorithms benefit from the improvement in signal to noise ratio provided by (i).

1. INTRODUCTION

Spatially variant apodization1 was developed in the context of synthetic aperture radar (SAR) imagery [1] as a means of adaptively controlling the window function used with a two dimensional fast Fourier transform (FFT) to form the image. Essentially it is a computationally efficient form of the minimum variance spectral estimator (Capon beamformer) [2]. It’s subsequent development, along with a fuller set of references, are documented in [3]. It seems to have been somewhat ignored by the signal processing community, which is surprising since it provides an answer to the classic problem associated with periodogram spectral analysis, i.e. which window should be used in a given application? SVA is adaptive in two senses: (i) the window is data dependent; (ii) the window varies with frequency.

Multi-coset sampling [4] is one form of compressive sampling. Essentially it consists of a number of sub-Nyquist A/D converters operating in parallel on the same analogue input. They all use exactly the same sampling frequency but the sampling instants are displaced in time across one sampling period. The output of each of these convertors is thus a conventional uniformly sampled signal. The sampled signal may however be heavily aliased because of the inadequacy of this uniform sampling rate with respect to the bandwidth of the analogue signal.

In this paper both SVA and multi-coset sampling are examined from the perspective of an airborne early warning (EW) system where spectral sensing is a significant requirement and computational complexity is severely limited. The initial motivation for considering SVA in this context is its superior resolution capability coupled with a modest increase in complexity with respect to a fixed window system for a conventional fully-sampled spectral sensing system. However, as demonstrated in Section 2, SVA also offers superior detection performance in Gaussian white noise, overcoming the classic window loss associated with the use of non-rectangular windows [5]. The reasons for examining a multi-coset method as a mechanism for sparse or compressive sensing in an EW receiver are: (i) it utilizes conventional A/D components; (ii) the uniformly sampled parallel channels give some hope that solutions can be found that are based on traditional FFT techniques. Counter arguments to the use of multi-coset sampling can be found in [6].

The development of multi-coset sampling of [4] is re-visited in section 3 (from a discrete-time perspective) to emphasize the concepts of folded-frequency and foldover number and to demonstrate that the identification of the spectral support associated with a sparse signal in noise can be viewed as a combination of periodogram spectral analysis on the uniformly sampled parallel channels combined with classical bearing estimation across the channels. Depending on the degree of frequency-domain sparsity associated with the analogue signal, the folded spectra associated with the parallel channels may be congested. Thus, in section 4, a multi-realization version of SVA is proposed to process the parallel uniformly-sub-sampled channels from the analogue to digital converters (ADC’s). The resultant folded high-resolution spectrum is used to make initial detection decisions, i.e. a frequency-ambiguous identification of the spectral support. The spectrum is then unfolded in the vicinity of these detections using a standard compressive sensing (CS) greedy algorithm known as orthogonalized least squares (OLS) [7] to identify foldover numbers. Section 5 considers options for reconstruction of the signal and/or the spectrum and section 6 provides some illustrative examples.

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2 for apodization read window; for spatially variant read frequency variant.
2. SVA AS A DETECTOR

Consider an $N$-point discrete Fourier transform (DFT):

$$X(n) = \sum_{k=0}^{N-1} x(k)W_N^{-nk}$$

where $W_N = e^{j2\pi/N}$ and the frequency bins are numbered $n = 0, 1, ..., N - 1$. Multiplying the data by an $N$-point positive-real valued window $\{w_k\}_{k=0}^{N-1}$, gives a windowed DFT:

$$X_w(n) = \sum_{k=0}^{N-1} w_k x(k)W_N^{-nk}$$

Consider a raised cosine window of the form:

$$w_k = 1 - \alpha \cos \left( \frac{2\pi k}{N} \right)$$

where: $0 \leq \alpha \leq 1$. When $\alpha = 0$, we have a rectangular window. When $\alpha = 1$ we have a Hamming window. The window parameter $\alpha$ allows us to control the amount of windowing applied in a simple but crude manner - we select a single parameter rather than a set of $N$ values for the window. Increasing $\alpha$ reduces the sidelobes of the frequency response at the expense of widening the main lobe.

This window can also be conveniently applied in the frequency domain because its DFT has only 3 non-zero terms. Given this formulation it is now possible to change the window with frequency by making $\alpha$ a frequency dependent:

$$X_w(n) = X(n) - \alpha(n) \left\{ \frac{X(n+1) + X(n-1)}{2} \right\}$$

Thus we have a spatially (frequency) variant windowing technique. Why would we want to do this? Consider a signal consisting of two large equally-powered closely-spaced frequencies at low frequency and a small high frequency component. To resolve the low frequency components a rectangular window would be best ($\alpha = 0$ because the main lobe of the frequency response of the window is as narrow as possible). The high sidelobes of the rectangular window does not matter since the high frequency component is small anyway. For the high frequency component, the main sources of interference are the two large frequencies far away from it. The best window for this high frequency component is one that makes the sidelobes of the frequency response of the window as small as possible i.e.: $\alpha = 1$. How then do we select the window parameter adaptively? One solution is the minimum variance spectral estimator [2].

The DFT can be viewed as a bank of filters each centred at a frequency indexed by $n$ the bin number. Looking at equation (2) the impulse response$^2$ of the filter at frequency bin $n$ is: $w_kW_N^{-nk}$. When a single frequency component, centered at this bin, i.e. $x(k) = W_N^{nk}$ is applied to the filter the output is:

$$X_w(n) = \sum_{k=0}^{N-1} \{W_N^{-nk}\} w_kW_N^{-nk} = \sum_{k=0}^{N-1} w_k$$

For the raised cosine window of (3) this is a constant, that is independent of the choice of $\alpha$, specifically $\sum_{k=0}^{N-1} w_k = N$ Thus for a raised cosine window, each filter in the filter-bank satisfies the distortion-less response condition required by a minimum variance spectral estimator. In order to produce a minimum variance estimate all we need to do is choose $\alpha(n)$ to minimize the variance $E[|X_w(n)|^2]$ at the output of each filter. For the single DFT of (1) we have no access to an ensemble, so we adopt a similar strategy to that taken in the development of the least mean squares (LMS) adaptive filter algorithm [8]: we replace the expectation with a single sample estimate $|X_w(n)|^2$. Thus, to find the optimum value of $\alpha$, we solve

$$\frac{d}{d\alpha} (|X_w(n)|^2) = 0$$

to give

$$\alpha(n) = \frac{2R\{X^*(n)\{X(n+1) + X(n-1)\})}{|X(n+1) + X(n-1)|^2}$$

While this equation is guaranteed to give a real value for $\alpha$, the values will not always be in the prescribed range between 0 and 1. If they are not within the range, we no longer have a raised cosine window. The solution is simply to restrict $\alpha$ to this range: if $\alpha(n) > 1$, $\alpha(n) = 1$ and if $\alpha(n) < 0$, $\alpha(n) = 0$. With these restrictions, equations (1), (4) and (6) define the SVA algorithm [1].

The detection performance of the fixed-window DFT of (2) is contrasted with that of the SVA for two different scenario’s in Fig’s 1 and 2. Both were conducted with a probability of false alarm $P_f = 0.01$ and with results averaged over $1 \times 10^5$ runs per data point. Figure 1 shows the probability of detection $P_d$ against output signal to noise ratio, $\text{SNR}_{\text{out}} = \frac{A^2}{2\sigma^2}$, where $A$ is the amplitude of the input phasor and $\sigma$ is the variance of the noise. The results indicate that the SVA performs nearly as well as a rectangular window and recovers the loss in performance associated with the other two common windows. Fig 2 demonstrates the advantage of the SVA detector in scenarios where a desired phasor of amplitude $A_{\text{int}}$ is close to an interfering one of amplitude $A_{\text{int}}$. The signal to interference ratio is defined as $\text{SIR}_{\text{out}} = \frac{A_{\text{int}}^2}{2\sigma_{\text{int}}^2}$ whilst the noise floor is 15 dB below the interference. The two phasors are separated by an average of 2 frequency bins distributed uniformly between 1 and 3 bins. The performance of the SVA is better than any of the fixed window systems.

3. MULTI-COSET SAMPLING

Consider a discrete-time signal $x(n)$, where $n$ indicates the sample number associated with a conventional A/D convertor operating at a desired sampling rate, $f_s$ Hz. Assume that the analogue signal has been suitably filtered prior to sampling to avoid aliasing of the signal or noise at this sampling rate. Conventional spectral analysis of this signal is usually based on the discrete Fourier transform (DFT) $X(k)$ of (1) and it’s windowed form $X_w(k)$, c.f. (2). To mimic the operation of a multi-coset sampler [4], re-write the signal $x(n)$ as $P$ parallel signals such as $x(n_1P+n_0)$, where $n_0 = 0, 1, ..., P-1$ is the sampling phase number (fast time), $M = N/P$ is the of number of samples in each of the parallel signals and $n_1 = 0, 1, ..., P-1$ is the sample number in each of the parallel channels (slow time). Each of the parallel channels is now sampling at $\frac{f_s}{P}$ Hz and will contain aliased components. In multi-coset sampling only $P_0 < P$ of the parallel channels are implemented to give an average sampling rate of $\frac{f_s}{P_0}$ Hz. For the moment, it will be assumed that all $P$ channels are available. This restriction will be removed in the course of the
development. Start by rewriting (1) in terms of the parallel channels.

\[
X(k) = \sum_{n_1=0}^{M-1} \sum_{n_0=0}^{P-1} x(n_1 P + n_0) W_{MP}^{-(n_1 P + n_0) k} \\
= \sum_{n_0=0}^{P-1} W_{MP}^{-n_0 k} \sum_{n_1=0}^{M-1} x(n_1 P + n_0) W_{M}^{-n_1 k}
\]

(7)

Subdividing the time index \( n \) into \( M \) \( P \)-sample segments, naturally subdivides the frequency index \( k \) into \( P \) \( M \)-sample segments Let \( k = k_1 M + k_0 \), where \( k_0 = 0, 1, \ldots, M-1 \) is the aliased frequency index associated with the slow time samples and \( k_1 = 0, 1, \ldots, P-1 \) is the fold-over number associated with aliasing in any of those channels. Substitution for \( k \) in (7) yields

\[
X(k_1 M + k_0) = \sum_{n_0=0}^{P-1} \left\{ W_N^{-n_0 k_0} X_M(k_0, n_0) \right\} W_P^{-n_0 k_1}
\]

(8)

where \( X_M(k_0, n_0) \) is the \( M \)-point DFT of the \( n_0 \)th sampling phase of \( x(n) \) - a slow time DFT, defined by

\[
X_M(k_0, n_0) = \sum_{n_1=0}^{M-1} x(n_1 P + n_0) W_M^{-n_1 k_0}
\]

(9)

Equation (8) has some similarities to a 2D DFT, i.e. an \( M \)-point DFT’s in slow-time \( X_M(k_0, n_0) \) followed by a \( P \)-point DFT \( \sum_{n_0=0}^{P-1} \{ \} W_P^{-n_0 k_0} \) in fast time of \( \{ \} \). However the phase correction term, \( W_N^{-n_0 k_0} \), which time shifts each of the slow-time DFT’s in fast time, couples the two directions, unlike the standard 2D DFT used in image processing. Equation (8) is in fact the basic building block for a radix-\( P \) decimation-in-time (DIT) fast Fourier transform (FFT) algorithm [9].

A key step in the application of compressive sensing is to construct a synthesis equation which describes the observed data as a linear combination of atoms (e.g. basis functions such as phasors). Equation (8) is key to this. Since it is a DFT it can be readily inverted using an IDFT. Thus

\[
\left\{ W_N^{-n_0 k_0} X_M(k_0, n_0) \right\} = \frac{1}{N} \sum_{k_1=0}^{P-1} X(k_1 M + k_0) W_P^{-n_0 k_1} \\
\triangleq \hat{X}_M(k_0, n_0)
\]

(10)

While \( \hat{X}_M(k_0, n_0) \) is not directly observed it can be readily calculated from the data using (9). As indicated earlier, in multi-coset sampling, only a set \( C \) containing \( P_0 \) of the \( P \) possible channels will be implemented, i.e. \( n_0 \in C \). Thus, even though \( X_M(k_0, n_0) \) can be calculated from the data using (9), the fast-time DFT of (8) cannot in general be used to calculate the desired Fourier coefficients \( X(k_1 M + k_0) \) since terms in the summation over \( n_0 \), fast-time, are missing.

Collecting together all \( P_0 \) equations such as (10) into a single vector equation:

\[
z(k_0) = W_{P_0 P} \begin{bmatrix} X(k_0) \\ X(M + k_0) \\ X((P-1)M + k_0) \end{bmatrix}
\]

= \( W_{P_0 P} x(k_0) \)

(11)
where the \((P_0 \times 1)\) vector \(\mathbf{z}(k_0)\) contains the terms \(X_M(k_0, n_0)\) that are available at folded frequency \(k_0\); the \((P \times 1)\) vector \(\mathbf{x}(k_0)\) contains the desired Fourier coefficients at folded frequency \(k_0\); the \((P_0 \times P)\) matrix \(\mathbf{W}_{P_0,P}\) is constructed from the associated scaled complex phasors \(W_{n_0}^{k_01}/N\)

Thus far the development has mirrored the original development of Feng & Bresler [4]. They proceed from this point by estimating the integrated covariance matrix: \(\mathbf{R}_s = \frac{1}{T} \sum_{k_0=0}^{M-1} \mathbb{E}[\mathbf{z}(k_0) \mathbf{z}^H(k_0)]\)
Eigenvalue decomposition of this matrix identifies which of the fold-over frequency bands are active. Hence the implied sparsity assumption is that a small number \((< P_0)\) of the fold over frequency bands are active. They show that estimation of the integrated covariance matrix and the subsequent signal reconstruction can be performed in the time-domain. However their method also requires interpolation of each of the channels for both the estimation and the reconstruction. The motivation for their method is to develop a multi-coset A/D converter that produces reconstructed output data at the intended sampling frequency. However in this paper the primary objective is spectral sensing. Having computed the FFT’s of \((13)\) these can also be used to form a frequency domain estimate of the integrated covariance matrix i.e. \(\mathbf{R}_s = \frac{1}{T} \sum_{k_0=0}^{M-1} \mathbf{z}(k_0) \mathbf{z}^H(k_0).\)
Given such a covariance matrix, identification of the active fold-over bands is equivalent to a bearing estimation problem on a non-uniform array given a set of snapshot vectors \(\{\mathbf{z}(k_0)\}_{k_0=0}^{M-1}\). This equivalence suggests that the eigenvalue decomposition (MUSIC algorithm) of \([4]\) may be inferior to maximum likelihood solutions particularly when the no. of snapshots is limited and signal/noise conditions are poor \([10]\). Approximate maximum likelihood algorithms such as \([11]\) which iterative recover the sources (fold-over numbers) may be particularly appropriate here. It is also worth noting that for spectral sensing applications, and for suitably chosen coset \(C\), the power profile of the signal as a function of fold-over number can be estimated from using an estimate of \(\mathbf{R}_s\) using least squares techniques when the number of active bands is greater than \(P_0\) \([12]\).

4. A FREQUENCY DOMAIN APPROACH

The approach, pursued here, starts from the assumption that the signal is sparse in the sense that it is made up of multiple narrowband components that can simultaneously occupy most fold-over bands but whose individual bandwidths are fractions of the folded bandwidth. Since spectrum analysis is a fundamental system requirement to enable detection of the presence of these components in the signal, it would be attractive to use it also to mitigate the effects of the multi-coset sampling. In particular the slow-time samples in each parallel channel are directly amenable to classical FFT-based spectrum analysis. For a stationary random process \(x(n)\), the folded power spectral density can be estimated from the \(P_0 < P\) multiple realizations over fast-time, \(n_0\):

\[
\hat{S}_{xx}(k_0) = \frac{1}{P_0} \sum_{n_0 \in C} |X_w(k_0, n_0)|^2
\]

where, as in \((4)\),

\[
X_w(k_0, n_0) = X_M(k_0, n_0) - \frac{\alpha(k_0)}{2} X_{av}(k_0, n_0)
\]

and

\[
X_{av}(k_0, n_0) = X_M(k_0 + 1, n_0) + X_M(k_0 - 1, n_0)
\]

Since \(M \gg 1\), the temporal aperture for the folded spectral estimation, \(P(M-1)\), is almost the same as conventional processing where the aperture is \(MP\), it is tempting to conclude that the frequency resolution is the same. However the sub-sampling leads to noise folding \([13]\) which degrades the resolution capability. The folded spectrum will be, by definition, more congested than the unfolded one. Thus it is desirable to maintain as good a resolution capability as possible. The SVA algorithm is a simple computationally inexpensive addition to standard FFT processing that has better resolution capability and detection performance than a fixed-window FFT. It is straightforward to derive a form of the algorithm that can be used to estimate the window parameter in \((13)\) and which takes advantage of the availability of multiple realizations of the same signal.

\[
\alpha(k_0) = \frac{2 \sum_{n_0 \in C} \mathbb{R}\{X_M(k_0, n_0)X_{av}(k_0, n_0)\}}{\sum_{n_0 \in C} |X_{av}(k_0, n_0)|^2}
\]

This folded spectrum can be used to detect signal components at particular folded frequency bins \(k_0\) in a straightforward way (in contrast to the eigenvalue decomposition and rank determination required to identify the fold-over numbers in \([4]\)). The detection process will thus identify the subset of folded frequencies that are active.

For single tones, the processing gain associated with \((12)\) is made up of two elements: (i) a coherent gain of \(M\) through the DFT (ii) a noncoherent gain of approximately \(\sqrt{P_0}\) from the post-FFT averaging. This is in contrast to a fully-implemented CS system with a processing gain of \(MP_0\) and a conventional Nyquist-rate system with a processing gain of \(MP\). It may be possible to recover the loss with respect to the former in the the following estimation step whose primary function is to estimate the fold-over number. This step is coherent since it exploits the relative phases of the parallel channels (in contrast to \((12)\)).

At each active folded frequency \(k_0\), equation \((10)\) must be solved to determine the fold-over number \(k_1\). This is a under-determined set of simultaneous equations, i.e.: more unknowns than equations. As such it is a classic (but small) compressive sensing problem. Possible solutions are based on the assumption that the set of Fourier coefficient \(\{X(k_1M + k_0)\}_{k_1=0}^{P-1}\) is sparse, i.e.: most of the coefficients are zero. The orthogonal least squares (OLS) algorithm \([7]\) is one such solution; an example of a greedy algorithm. An example of it’s use in compressive sensing problems is described in \([14]\). By contrast, the simplest solution is to: (i) use a \(P\)-point DFT and effectively assume that the data in the missing parallel channels is zero; (ii) search for the maximum at the DFT output. This is the optimum solution, in a maximum likelihood (ML) sense, if only one fold-over number is present at a particular folded frequency. It is also the first step in the OLS algorithm. Note also that since \(k_1\) is naturally an integer, the search space for the OLS algorithm is restricted to \(P\) atoms, i.e. the columns of \(\mathbf{W}_{P_0,P}\).

5. ESTIMATION & RECONSTRUCTION

Having identified frequency pairs \((k_1, k_0)\) where the signal is active, there are a number of ways to estimate the spectrum (and reconstructed the signal if it is required). The simplest is to accept the values of Fourier coefficients \(X(k_1M + k_0)\) at the active frequencies, use these as the estimate of the spectrum, i.e. \(\{X(k_1M + k_0)\}^2\), and use the inverse of \((1)\) to reconstruct the signal at the full sampling rate \(f_s\) Hz.

\[
x(n) = \frac{1}{N} \sum_{k \in S} X(k) W_{N}^{nk}
\]

where \(S\) is the set of unfolded frequencies that correspond to the pairs \((k_1, k_0)\). A more robust method is to use least squares (LS)
to reconstruct the observed $MP_0$ samples from the active frequency components. A synthesis equation can be constructed from (16)

$$x_{mc} = W_{mc,a}X_a$$

(17)

where $x_{mc}$ is an $(MP_0 \times 1)$ vector containing the multi-coset samples of $x(n)$; $X_a$ is an $(Na \times 1)$ vector containing the $Na$ active frequency components of the Fourier coefficients $X(k)$. The $(MP_0 \times Na)$ matrix $W_{mc,a}$ is made up of the appropriate phasors such as $W_{c,k}^n$. The scale factor $1/N$ can be absorbed into Fourier coefficients. The LS equation (17) can be solved in a number of ways. Numerically, Householder reflections or Givens rotations \([46,616]\) are attractive. The latter had been suggested for similar real-time radar signal processing problems and maps well to fixed-point implementations. The analytic solution is:

$$\hat{X}_a = \left(W_{mc,a}^H W_{mc,a}^{-1}\right)^{-1} W_{mc,a}^H x_{mc}$$

(18)

Given $\hat{X}_a$, the signal can be reconstructed using the IDFT of (16).

### 6. EXAMPLES

The test signal considered here is complex. The total number of samples is $N = 784$ which is made up of a possible $P = 6$ channels, each with access to $M = 64$ slow time samples. The signal consists of 4 complex phasors whose frequencies are chosen at random and have power levels of -20 dB, -20 dB, 0 dB and 0 dB. There is also a coloured noise component whose bandwidth is 0.08 of the folded frequency bandwidth - this is to mimic a communications signal. The thermal noise level is set at -30 dB. The number of multi-coset channels is $P_0 = 3$ with sampling points $n_0 \in C = \{0, 2, 3\}$. The average sampling rate is thus $f_s/2$ Hz. Fig 3 illustrates the results. As a point of reference, a straightforward spectral analysis of all $N = 784$ samples, using a Hamming window and a DFT zero-padded by a factor of 2 is shown in panel (a). The frequency scale is normalized to the fold over frequency $f_s/P$, i.e. the normalised frequency scale is:

$$\frac{k}{M} = \frac{k_1 + k_0}{M}$$

(19)

where $0 \leq \frac{k}{M} < P$. The centre frequency for the coloured noise is placed at 4 on this scale. The four phasors are easily identifiable.

Panel (b) shows the output of the multi-realisation SVA spectral estimate of 12, averaged over the $P_0 = 3$ slow time channels. The effects of aliasing are clearly visible. The black arrow indicates where one of the phasors has aliasied to. Also of note is the coloured noise. Components to the right of normalised frequency 4 in panel (a) are present just above folded frequency zero in (b). Components to the left of 4 now appear just below 1 on (b). SVA is particularly advantageous here as the folded spectrum of (b) may be crowded as all the significant components fold onto the range between 0 and 1. The nominal noise level present in this spectrum is -50 dB. A detection threshold of -45 dB is used to identify “significant components”. These are indicated by the black circles. Thus, despite aliasing, it is still possible to perform the detection function. If nothing is detected, no further processing is required, saving in computational resources and/or power.

Equation (8) is then used to estimate $X(k_1 M + k_0)$ at the folded frequencies $k_0$ identified by the detection process. This is performed using a $P$-point DFT at each folded frequency where a component has been detected in (b).

$$\hat{X}(k_1 M + k_0) = \sum_{n_0 \in C} \left(W_{c,k_0}^{-n_0} X_w(k_0, n_0)\right) W_{c,k_1}^{-n_0}$$

The unknown values of $X_w(k_0, n_0)$, i.e. $n_0 \in \hat{C}$, are simply set to zero. As noted in section 4, this is the first step in the OLS algorithm [7]. It is sufficient if only one fold-over number is active at a particular folded frequency. Thus the detection process in (b) reduces the number of $P$-point DFT’s required. The magnitude of the estimated Fourier coefficients are displayed in panel (c) with the normalised folded frequency $k_0/M$ on the horizontal axis and the fold-over number $k_1$ on the vertical axis. The column identified by the arrow indicates the response to one of the complex phasor components detected in panel (b). The maximum response (in red) is found by searching over all fold-over numbers at this particular folded frequency. The result of the search is indicated by the white circle. Careful examination of this particular column reveals two significant sidelobes at $k_1 = 0$ and $k_1 = 4$ that are a direct result of non-uniform sampling in fast-time.

The white circles indicate the estimated significant components in the signal. This is displayed more conventionally in panel (d) as the black line. As most of the Fourier coefficients are zero (and they are displayed in dB) these zero values are set to the detection threshold of -45 dB. Finally the LS method of section 5 is used to reconstruct the signal at the full sampling rate. This reconstructed signal is then analyzed using exactly the same method as used for panel (a) to give the red curve.

An additional set of results is illustrated in Figure 4 for $P = 11$, $M = 64$ and $n_0 \in \hat{C} = \{0, 5, 7, 8\}$. The average sampling rate is now $\frac{4}{11} f_s$. The coloured noise component is again centred at a normalised frequency of 4 and there are four phasors as before at random frequencies. As example of the limitations of the technique are highlighted by the arrow. The component at 8 is folded to be coincident with the coloured noise in (b). The peak detector select the larger component, leaving the smaller component to be set to zero. This leave a null in the middle of coloured noise spectrum when it is unfolded in (d). It is only reconstituted by the smearing associated with the DFT analysis of the reconstructed signal. The full OLS algorithm may go some way to overcoming this problem.
7. CONCLUSIONS

The potential benefits of using SVA rather than a fixed-window FFT processing for a conventional EW spectral sensing system have been demonstrated. Simple solutions to adding a compressive sensing capability to such a system have been explored through the use of multi-coset sampling techniques to reduce the average sampling rate. The parallel uniformly-sampled channels are viewed as a resource with which to detect the presence of signals and as a first step in identifying their spectral support. A multi-realisation version of SVA is proposed to alleviate the congestion in the resultant folded spectrum. Once the active components in the folded spectrum have been identified, the foldover numbers associated with them can be identified using a greedy algorithm such as OLS. The latter benefits from the improved signal to noise ratio provided by the use of the SVA. It is also noted that the first step of the OLS algorithm for this problem is itself a DFT.

8. REFERENCES


